Int J Theor Phys (2008) 47: 1594–1599
DOI 10.1007/s10773-007-9599-4

On Axially Symmetric Domain Walls and Cosmic Strings in Bimetric Theory

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Received: 12 August 2007 / Accepted: 19 October 2007 / Published online: 6 November 2007

Abstract It is shown that the axially symmetric thick domain walls and cosmic strings do not survive in the frame work of bimetric theory of gravitation proposed by Rosen (Gen. Relativ. Gravit. 4:435, 1973). Hence vacuum models are presented and studied.

Keywords Domain walls · Cosmic strings · Bimetric theory

1 Introduction

The large scale structure of the universe has been an active field of investigation. The phase transitions at the early stages of evolution of the universe can have important cosmological consequences, in particular, they can give rise to vacuum domain structures [5, 7, 22, 24]. The three possible structures are vacuum domain walls, strings and monopoles which may, possibly have survived to the present day. In recent years there has been a lot of interest in the study of domain walls and cosmic strings because of the fact that the existence of domain walls and a large scale network of cosmic strings does not contradict observation. Also they are important in the formation of galaxies and large scale structure of the universe [6, 20, 21].

A considerable amount of work has been done on domain walls and cosmic strings in general relativity and in alternative theories of gravitation. In particular Reddy [13, 14], Rahaman et al. [12], Pradhan et al. [10] and Rahaman and Mukherji [11] are some of the

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authors who have investigated various aspects of domain walls and cosmic strings in alternative theories of gravitation.

Rosen [19] formulated a bimetric theory of gravitation which satisfies the principles of covariance and equivalence. He has modified the formalism of the general relativity theory introducing into it, besides the metric tensor g_{ij} associated with the line element

$$ds^2 = g_{ij} dx^i dx^j$$

a second metric tensor corresponding to flat space-time described by the line element.

$$d\sigma^2 = \gamma_{ij} dx^i dx^j$$

at each point of the space-time. The tensor g_{ij} describes gravitation and interacts with matter. The back ground metric γ_{ij} refers to inertial forces. One can regard γ_{ij} as giving the geometry that would exist if there were no matter. This theory also agrees with the present day observational facts of general relativity theory. Using the conventional variational principle Rosen [19] obtained the field equations in bimetric theory of gravitation which read as

$$N_{ij} - \frac{1}{2} g_{ij} N = -8\pi k T_{ij} \tag{1}$$

where

$$N^i_j = \frac{1}{2}\gamma^{ab}(g^{hi} g_{hi}|_a)|_b$$

and

$$N = N^i_i, \qquad g = \det(g_{ij}), \qquad k = \left(\frac{g}{y}\right)^{1/2}$$

in which T_{ij} is the usual stress tensor of matter and a vertical bar (|) indicates covariant differentiation with respect to γ_{ij}.

With the advent of new theories of gravitation, it has been a common practice to investigate several aspects of the new theories and then to compare with the results of Einstein's theory of gravitation. With this motivation a lot of work has been done in Rosen's bimetric theory of gravitation [4, 16, 18, 23]. In particular Mohanty et al. [9] and Mohanty and Sahoo [8] have shown the non-existence of perfect fluid or mesonic perfect fluid models in bimetric theory while Reddy [15] has established the non-existence of Bianchi type string cosmologies in this theory. Recently, Reddy et al. [17] have shown that plane symmetric domain walls and cosmic strings do not exist in bimetric theory. Inspite of the fact that a lot of work has been done, in this direction, it is evident from literature that there is need for further investigation which may unravel some of the hidden secrets of the theory.

In this paper we show that axially symmetric domain walls and the cosmic strings in bimetric theory of gravitation do not survive.

2 Domain Walls

A thick domain wall can be viewed as a soliton like solution of the scalar field equation coupled with gravity. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field ϕ with self interactions contained in a potential

$V(\phi)$ given by

$$T_{ij} = \phi,_i \phi,_j - g_{ij}\left(\frac{1}{2}\phi,_k \phi^{,k} - V(\phi)\right) \tag{2}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + w_i w_j) + p w_i w_j, \quad w_i w^j = -1 \tag{3}$$

where ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction. Chatterji et al. [3] have obtained an exact solution for a time dependent metric and a scalar field for a thick domain wall in Brans–Dicke [2] theory of gravitation. Here, we use the second approach and show that axially symmetric thick domain walls do not survive in the frame work of Rosen's bimetric theory of gravity.

We consider the axially symmetric metric given by [1]

$$ds^2 = dt^2 - a^2[dx^2 + f^2(\chi)d\phi^2] - b^2 dz^2 \tag{4}$$

with the convention $x^1 = x$, $x^2 = \phi$, $x^3 = z$, $x^4 = t$ and a and b are functions of z and the proper time t while f is a function of the coordinate χ alone. The flat space–time corresponding to the metric (4) is

$$d\sigma^2 = dt^2 - dx^2 - d\phi^2 - dz^2 \tag{5}$$

In the commoving coordinate system we have from (3)

$$T_0^0 = T_1^1 = T_2^2 = \rho, \qquad T_3^3 = -p, \qquad T_j^i = 0, \quad i \neq j \tag{6}$$

(here pressure is taken in the direction of z-axis). The quantities ρ and p depend on z and t only.

The field equations (1) of Rosen's bimetric theory of gravity with the help of (4–6), now, take the form

$$\begin{aligned}
&\frac{1}{2}\left(\frac{f_1}{f}\right)_1 + \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = -8\pi\rho \\
&\frac{1}{2}\left(\frac{f_1}{f}\right)_1 + \left(\frac{a_3}{a}\right)_3 + \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \left(\frac{a_4}{a}\right)_4 + \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = -8\pi\rho \\
&\frac{1}{2}\left(\frac{f_1}{f}\right)_1 + \left(\frac{a_3}{a}\right)_3 + \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \left(\frac{a_4}{a}\right)_4 - \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = -8\pi\rho
\end{aligned} \tag{7}$$

where the suffixes 1, 3 and 4 hereafter, denote differentiation with respect x, z and t respectively.

The functional dependence of the metric (4) together with (7) imply [1]

$$\left(\frac{f_1}{f}\right)_1 = k^2, \quad k^2 = \text{constant} \tag{8}$$

If $k = 0$, then $f(\chi) = (\text{const})\chi$, $0 < \chi < a$. This constant can be made equal to 1 by suitably choosing units for ϕ. Thus we shall have

$$f(\chi) = \chi \tag{9}$$

Now the field equations (7) reduce to

$$\frac{1}{2}\left[\left(\frac{b_3}{b}\right)_3 - \left(\frac{b_4}{b}\right)_4\right] = -8\pi\rho \tag{10}$$

$$\left(\frac{a_3}{a}\right)_3 - \left(\frac{a_4}{a}\right)_4 - \frac{1}{2}\left[\left(\frac{b_3}{b}\right)_3 - \left(\frac{b_4}{b}\right)_4\right] = 8\pi\rho \tag{11}$$

$$\left(\frac{a_3}{a}\right)_3 - \left(\frac{a_4}{a}\right)_4 + \frac{1}{2}\left[\left(\frac{b_3}{b}\right)_3 - \left(\frac{b_4}{b}\right)_4\right] = -8\pi\rho \tag{12}$$

To solve the filed equations (10–12), we note that there are three equations connecting four unknowns, a, b, p and ρ. So one relation connecting these variables is needed. Here we assume the relation between the metric coefficients such as [1]

$$a = \alpha b, \quad \alpha \neq 0 \text{ is a constant} \tag{13}$$

The field equations (10) and (12) yield

$$\left(\frac{a_3}{a}\right)_3 - \left(\frac{a_4}{a}\right)_4 = 0 \tag{14}$$

By using the method of separation of variables, (14) gives us the solution.

$$a(z,t) = \exp\left[\frac{k}{2}(z^2 + t^2) + k_1 z + k_2 t\right] \tag{15}$$

Then, in view of (13), we have

$$b(z,t) = \alpha \exp\left[\frac{k}{2}(z^2 + t^2) + k_1 z + k_2 t\right] \tag{16}$$

where k, k_1, and k_2 are constants. Use of equations (13–16) in equations (10–12) will, now, yield

$$\rho = p = 0 \tag{17}$$

which shows that axially symmetric thick domain walls do not survive in Rosen's bimetric theory of gravitation.

When $\rho = p = 0$ (vacuum), using (9, 15) and (16) in (4), axially symmetric vacuum model in Rosen's bimetric theory can be written as

$$\begin{aligned} ds^2 = dt^2 &- \exp[k(z^2 + t^2) + 2(k_1 z + k_2 t)][dx^2 + \exp(2\chi)d\phi^2] \\ &- \alpha^2 \exp[k(z^2 + t^2) + 2(k_1 z + k_2 t)]dz^2 \end{aligned} \tag{18}$$

The model (18) has no singularities either at $z = 0$ or at $t = 0$ and it does have singularities as $z \to \infty$ and $t \to \infty$. The spatial volume of the model is given by

$$V^3 = (-g)^{1/2} = \alpha \exp\left[\frac{3}{2}k(z^2 + t^2) + 3(k_1 z + k_2 T) + \chi\right] \tag{19}$$

which shows that the model is expanding with time t and z since $\alpha > 0$ and $0 < \chi < \infty$.

3 Cosmic Strings

In this section we establish that the axially symmetric cosmic strings, which have received considerable attention in general relativistic cosmology, do not exist in the frame work of bimetric theory of gravitation proposed by Rosen [19].

We consider the axially symmetric cosmic string dust source with energy momentum tensor [6]

$$T_j^i = \rho u^i u_j - \lambda x^i x_j \tag{20}$$

where u^i is the four velocity of the string cloud, x^i is the normal space-like four-vector, ρ and λ are the rest energy density of the cloud of strings and the tension density of the string cloud respectively. The string source is along the z-axis, which is the axis of symmetry.

We consider the axially symmetric metric given by (4). Orthonormalisation of u^i and x^i, is given as

$$u^i u_i = 1, \qquad u^i x_i = 0, \qquad x^i x_i = -1 \tag{21}$$

Adopting the comoving coordinate system, we have from (20),

$$T_4^4 = \rho, \qquad T_1^1 = 0, \qquad T_2^2 = 0, \qquad T_3^3 = \lambda, \qquad T_j^i = 0 \quad \text{for } i \neq j \tag{22}$$

The quantities ρ and λ depend on z and t.

Rosen's bimetric field equations (1) for the axially symmetric metric (4) with the help of (5, 20, 21) and (22) take the form

$$\begin{aligned}
&\frac{1}{2}\left(\frac{f_1}{f}\right)_1 + \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \frac{1}{2}\left(\frac{b_3}{b}\right)_4 = 0\\
&\frac{1}{2}\left(\frac{f_1}{f}\right)_1 + \left(\frac{a_3}{a}\right)_3 - \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \left(\frac{a_4}{a}\right)_4 + \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = 8\pi\lambda\\
&\frac{1}{2}\left(\frac{f_1}{f}\right)_1 + \left(\frac{a_3}{a}\right)_3 + \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \left(\frac{a_4}{a}\right)_4 - \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = -8\pi\lambda
\end{aligned} \tag{23}$$

Which, in view of the similar argument as in the case of domain walls, reduce to

$$\left(\frac{b_3}{b}\right)_3 - \left(\frac{b_4}{b}\right)_4 = 0 \tag{24}$$

$$\left(\frac{a_3}{a}\right)_3 - \left(\frac{a_4}{a}\right)_4 - \frac{1}{2}\left(\frac{b_3}{b}\right)_3 + \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = -8\pi\lambda \tag{25}$$

$$\left(\frac{a_3}{a}\right)_3 - \left(\frac{a_4}{a}\right)_4 + \frac{1}{2}\left(\frac{b_3}{b}\right)_3 - \frac{1}{2}\left(\frac{b_4}{b}\right)_4 = -8\pi\rho \tag{26}$$

Here, again, we have three field equations in four unknowns. Hence assuming the same relation, between metric coefficients, as given by (13), we arrive at the same solution given by (15) and (16) with

$$\rho = \lambda = 0$$

which in turn yield the same vacuum model in Rosen's theory given by (18). Thus we conclude that in bimetric theory of gravitation the axially symmetric cosmic strings do not survive.

4 Conclusions

It is well known that at the early stage of evolution of universe topologically stable objects such as domain walls and cosmic strings play a fundamental role in the formation of universe. It is evident, from the literature, that Einstein's formalism of general relativity has been extensively used to establish the existence of thick domain walls and cosmic strings. Recently, Reddy et al. [17] have shown the non-existence of plane symmetric domain walls and cosmic strings in Rosen's [19] bimetric theory of gravitation. Here we have shown that axially symmetric thick domain walls and cosmic strings do not survive in Rosen's bimetric theory of gravitation. Hence, we can conclude that bimetric theory doesn't help in any way to study the gravitational effects of thick domain walls and cosmic strings at the early stages of evolution of the universe.

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